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Non-decoupling Effects of Heavy Particles in Triple Gauge Boson Vertices

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Abstract

Non-decoupling effects of heavy particles present in beyond-the-standard models are studied for the triple gauge boson vertices γW^+W^- and $Z^0W^+W^-$. We show from a general argument that the non-decoupling effects are described by four independent parameters, in comparison with the three parameters S, T and U in the oblique corrections. These four parameters of the effective triple gauge boson vertices are computed in two beyond-the-standard models. We also study the relation of the four parameters to the S, T, U parameters, relying on an operator analysis.

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1. Introduction

Any unified model of electro-weak (and strong) interactions beyond the standard model is characterized by the existence of heavy particles of masses $M \gg M_W$, M_W being the weak-boson mass. Since such heavy particles are not likely to be discovered in the immediate future, it is of importance to ask how their effects may be detected in low energy ($E \lesssim M_W$) processes of light ($m \lesssim M_W$) particles, such as those measured at LEP and SLC experiments, through radiative corrections. In this paper we will restrict ourselves to those classes of beyond-the-standard models in which heavy particles manifest themselves in low energy processes only through loop effects.

It is useful to divide the loop effects of heavy particles (the new physics contributions) into two types: i) those which virtually decouple in the limit $M \to \infty$ and ii) those which do not decouple in the same limit.

In the type i) the heavy mass M is dominated by a new large mass scale M_S, $M^2 = M_S^2 + O(M_W^2)$. Note that the term giving M_S is $SU(2)_L \times U(1)$ singlet, and hence the heavy particle contributions to low-energy processes are suppressed by $1/M_S^2$ [1]. On the other hand, in the type ii) M has its origin in the $SU(2)_L \times U(1)$ breaking due to the VEV of the Higgs ϕ, and large M means a large coupling constant. The latter factor appearing in the numerator of amplitudes cancels the suppression factor $1/M^2$, leading to non-decoupling effects of heavy particles.

We will be concerned with non-decoupling effects of the type ii), which will provide us with some useful constraints on the properties of new physics which might lie beyond the standard model. As far as light fermion processes are concerned, we only have to deal with the non-decoupling contributions to the gauge boson two-point functions (oblique correction [2]). These corrections are summarized in terms of three parameters S, T and U[3-6]. A few implications on beyond-the-standard models have been obtained recently. For instance, realistic technicolor models have been shown to contradict the observed value of the S [3,6]. We have derived a bound on the number of possible heavy extra generations of fermions by a combined use of recent data on the S and T [7].

The next question to ask is whether similar non-decoupling contributions of heavy particles are present in higher n-point functions of gauge bosons. The answer is known. Namely, (at one-loop level) for $n > 4$ the coefficient of the relevant operator O_i is suppressed by $1/M^{n-4}$ or more strongly since O_i has dimension $d_i \geq n$. Hence $n =$

2, 3, 4 are the only possibility to have non-decoupling effects. In this paper we wish to study the non-decoupling effects in the triple gauge boson (TGB) vertices ($n = 3$). These vertices can be measured in the W^+W^- production at LEP200.

The purpose of the present paper is to identify the minimum set of parameters which describe the non-decoupling effects of heavy particles in the TGB vertices, just as the S, T and U parameters do in the oblique corrections. We will show by a general argument that the non-decoupling effects in the TGB vertices are summarized in terms of four parameters. This conclusion from a general analysis will be confirmed by the evaluation of the four parameters at one-loop level in two examples of beyond-the-standard model, (a) an extra fermion generation and (b) technihadrons.

We may easily understand why the non-decoupling effects in the TGB vertices are inevitable from a simple operator analysis. In a theory with spontaneous gauge symmetry breaking, quantum corrections due to heavy loops can be described in terms of gauge invariant effective operators consisting of the gauge fields and the Higgs field ϕ. Gauge non-invariant effective operators in the broken phase arise when ϕ is replaced by the sum of its VEV v and the shifted field, as exemplified by the operator of dimension 6 [8,9],

$$(\phi^\dagger \sigma^a \phi) W^a_{\mu\nu} B^{\mu\nu} = v^2 W^3_{\mu\nu} B^{\mu\nu} \quad , \tag{1}$$

where $W^3_{\mu\nu} = \partial_\mu W^3_\nu - \partial_\nu W^3_\mu - igW^+_\mu W^-_\nu$. The *r.h.s.* of eq.(1) is an expression in the broken symmetry phase. The first term of $W^3_{\mu\nu}$ contributes to the S parameter, while the second induces a TGB coupling, $W^+_\mu W^-_\nu B^{\mu\nu}$.

The question how the parameters describing the non-decoupling effects in the TGB couplings are related to the parameters S, T U depends on whether the new physics contributions in question are of the type i) or ii) mentioned above. This question will be answered later.

2. Triple gauge boson vertices and four parameters

In the present paper we consider two kinds of triple gauge boson (TGB) vertices VW^+W^- where V denotes photon γ or Z^0. These couplings can be measured by

observing the $W^\pm$ pair production in e^+e^- collisions,

$$e^- + e^+ \to W^- + W^+ \quad . \tag{2}$$

Both neutrino exchange in the t-channel and V exchange in the s-channel contribute to this process (Fig. 1). In the t-channel process the heavy particle effects are confined to the loop correction on the external $W^\pm$ legs. The results can be found in the literature [10], and we will not discuss them further. The V exchange diagram consists of three factors, the V propagator Π, the VW^+W^- vertices Γ^V and the external leg corrections Π'_{11} of $W^\pm$ (Fig.1). The heavy particle effects in the V propagator have already been taken into account as the oblique corrections. It suffices to consider the VW^+W^- vertices Γ^V.

We define the kinematics of the VW^+W^- vertex as

$$V(p,\ \epsilon_1) \to W^-(q,\ \epsilon_2) + W^+(\bar{q},\ \epsilon_3)\ , \tag{3}$$

where $p,\ q,\ \bar{q}$ are the momenta of $V,\ W^-,\ W^+$ respectively and $\epsilon_1,\ \epsilon_2,\ \epsilon_3$ their polarization vectors. The produced $W^\pm$ are on-shell and we impose

$$q^2 = \bar{q}^2 = M_W^2\ , \quad q\cdot\epsilon_2 = \bar{q}\cdot\epsilon_3 = 0 \quad . \tag{4}$$

We are interested in the low-energy process, which means that

$$p^2 = (q+\bar{q})^2\ , \quad q^2 = \bar{q}^2 \ll M^2 \quad . \tag{5}$$

The process (2) in the high energy limit, $p^2 \gg M^2$ has been studied in [10]. In many of the models with heavy particles, at one-loop level, the VW^+W^- vertex preserves CP invariance, which we assume in this article.

Let $g_{WWV}\ \Gamma^V_{\mu\alpha\beta}$ be the effective VW^+W^- vertex, where $g_{WW\gamma} = -e,\ \ g_{WWZ} = -e\cot\theta_W$. Following Hagiwara, Hikasa, Peccei and Zeppenfeld [11], we decompose the effective vertex into pieces with different Lorentz structures,

$$\begin{aligned}\Gamma^V_{\mu\alpha\beta} =& f_1^V (q-\bar{q})_\mu g_{\alpha\beta} + f_2^V (q-\bar{q})_\mu p_\alpha p_\beta + f_3^V (g_{\mu\beta}p_\alpha - g_{\mu\alpha}p_\beta) \\ &+ i f_5^V \epsilon_{\mu\alpha\beta\rho}(q-\bar{q})^\rho + i h^V p_\mu \epsilon_{\alpha\beta\rho\sigma} p^\rho (q-\bar{q})^\sigma\end{aligned} \tag{6}$$

(we take the convention $\epsilon^{0123} = 1$). We have imposed the on-shell conditions for $W^\pm$ and CP invariance. The terms proportional to p_μ have been ignored, since they give

terms proportional to external electron masses on using the equation of motion. The last term with h^V is redundant in this sense, but it is kept to make unbroken $U(1)_{em}$ gauge invariance manifest in the case of $V = \gamma$.

At the tree level f_1^V and f_3^V are the only non-vanishing form factors,

$$f_1^\gamma = f_1^Z = 1 \quad , \qquad f_3^\gamma = f_3^Z = 2 \quad . \tag{7}$$

This in turn means that the one-loop corrections to f_1^V and f_3^V are apparently ultraviolet-divergent. When the external leg correction, or the wave-function renormalization factor of $W^\pm$

$$Z_W = 1 + g^2 \Pi_{11}^{'} \ , \tag{8}$$

is combined, the divergent form factors are rendered finite. Namely,

$$\begin{aligned} f_1^V + g^2 \Pi_{11}^{'} &\equiv 1 + \bar{f}_1^V \ , \\ f_3^V + 2g^2 \Pi_{11}^{'} &\equiv 2 + \bar{f}_3^V \ , \end{aligned} \tag{9}$$

where $\bar{f}_i^V$ are the finite one-loop corrections to f_i^V.

This prescription to get finite form factors is somewhat different from that adopted in Ref.[10], where the $W^3\gamma$ self-energy Π_{3Q}^P is chosen, instead of $\Pi_{11}^{'}$, to get the finite form factors. The "charge universality" is manifest in our choice of form factors, i.e. $\bar{f}_1^\gamma = 0$ as will be discussed below, while $f_1^\gamma + g^2 \Pi_{3Q}^P \neq 1$.

We are now ready to show how many parameters are necessary to describe the non-decoupling effects in the VW^+W^- vertices. We follow the way of argument by Altarelli and Barbieri concerning the oblique corrections [5]. First we note that in the expansion of each form factor $f_i^V(p^2)$ in the powers of p^2/M^2, only the lowest term survives, since all form factors are at most dimensionless. Thus we are left with the parameters $f_i^V(0)$ as the candidates, which we will refer to simply as f_i^V hereafter. The parameter f_2^V actually decouples, since it has mass dimension -2 and therefore is suppressed by $1/M^2$.

Next we have to take account of the $U(1)_{em}$ gauge invariance. This amounts to imposing the current conservation, $p^\mu \Gamma^\gamma_{\mu\alpha\beta} = 0$. Under the on-shell condition for $W^\pm$,

only f_5^γ is subject to a non-trivial condition,

$$f_5^\gamma(p^2) = (-p^2)h^\gamma \quad . \tag{10}$$

Since h^γ has no singularity at $p^2 = 0$, we conclude that $f_5^\gamma(p^2)$ is suppressed by p^2/M^2 and decouples.

We should also consider the condition which arises as a result of "charge universality" of the photon coupling. In QED the electric charge defined in the Thomson limit $p^2 = 0$ has a universal meaning, i.e. irrespectively of external particles; it is fixed by Z_3, the wave-function renormalization of the photon. In the $SU(2)_L \times U(1)$ theory, the non-Abelian nature is inessential in this respect, and hence the electric charge is fixed by the oblique corrections alone; the vertex correction to f_1^γ and the external leg corrections of $W^\pm$ should cancel out in the "effective Thomson limit" $p^2 \ll M^2$, i.e.

$$\bar{f}_1^\gamma = f_1^\gamma(0) + g^2\Pi'_{11}(0) - 1 = 0 \quad . \tag{11}$$

To summarize, we have shown that there remain four independent parameters

$$\bar{f}_1^Z \quad , \bar{f}_3^\gamma \quad , \bar{f}_3^Z \quad , f_5^Z \tag{12}$$

in order to describe the non-decoupling loop corrections to the TGB couplings due to heavy particles.

Experimental constraints on the four parameters may be obtained from precise measurements of $e^- + e^+ \to W^- + W^+$ process. The four parameters participate in the s-channel matrix element. As for the loop corrections to the V propagator, the oblique corrections, they can be taken into account by replacing electric charge e, Weinberg angle s, and M_Z by their corresponding star quantities e_*, s_*, M_{Z^*} at the tree level amplitude [2,3,10]. In a similar way, the sum of remaining vertex correction and the external leg correction of $W^\pm$ can be taken account of by replacing $\Gamma^V_{\mu\alpha\beta}$ at the tree level by, say, $\Gamma^{*V}_{\mu\alpha\beta}$, where

$$\begin{aligned}
\Gamma^{*\gamma}_{\mu\alpha\beta} &= (q-\bar{q})_\mu g_{\alpha\beta} + (2+\bar{f}_3^\gamma)(g_{\mu\beta}p_\alpha - g_{\mu\alpha}p_\beta), \\
\Gamma^{*Z}_{\mu\alpha\beta} &= (1+\bar{f}_1^Z)(q-\bar{q})_\mu g_{\alpha\beta} + (2+\bar{f}_3^Z)(g_{\mu\beta}p_\alpha - g_{\mu\alpha}p_\beta) \\
&\quad + i f_5^Z \epsilon_{\mu\alpha\beta\rho}(q-\bar{q})^\rho \quad .
\end{aligned} \tag{13}$$

Thus the loop-corrected s-channel matrix element is given as

$$\begin{aligned} M = & - ie_*^2 Q(\bar{v}\gamma^\mu u)\frac{1}{p^2}\Gamma^{*\gamma}_{\mu\alpha\beta}\epsilon_2^\alpha(q)\epsilon_3^\beta(\bar{q}) \\ & - ie_*^2\frac{(I_3 - s_*^2 Q)}{s_*^2}(\bar{v}\gamma^\mu u)\frac{1}{p^2 - M_{Z^*}^2}\Gamma^{*Z}_{\mu\alpha\beta}\epsilon_2^\alpha(q)\epsilon_3^\beta(\bar{q}) \quad , \end{aligned} \tag{14}$$

where u and v are electron and positron (Weyl) spinors, and $\epsilon_2^\alpha(q)$ and $\epsilon_3^\beta(\bar{q})$ $W^\pm$ polarization vectors. In eq.(14), e^+ and e^- are assigned a definite helicity, and $I_3 = -\frac{1}{2}$ for e_L, $I_3 = 0$ for e_R $(Q = -1)$.

The four form factors get contributions from ordinary particles of the standard model as well. These contributions of the standard model [12] have to be subtracted from the measured values of the form factors, when one tries to derive experimental constraints on the non-decoupling effects of heavy particles.

3. New physics contributions to the four parameters

Here we evaluate the four non-decoupling parameters by taking two examples of new physics contributions: (a) an extra fermion generation and (b) technihadrons.

(a) *Extra fermion generation*

Consider an extra generation of quarks and leptons (Q and L):

$$\begin{pmatrix} U \\ D \end{pmatrix}_L \ , \quad U_R \ , \ D_R \ ; \begin{pmatrix} N \\ E \end{pmatrix}_L \ , \quad N_R \ , \quad E_R \ . \tag{15}$$

They are assumed to be a color triplet and a singlet (the color factor $N_Q = 3$ and $N_L = 1$) and to have the hypercharge $Y_Q = 1/6$ and $Y_L = -1/2$ (for the $SU(2)_L$ doublet), respectively. The neutral lepton N is assumed to have a Dirac mass. The fermions are all assumed to be heavy, $m_F^2 \gg M_W^2$. The one-loop graphs of the TGB vertices are ultraviolet-divergent and have to be regularized. We have computed the parameters $\bar{f}_1^Z$, $\bar{f}_3^\gamma$, $\bar{f}_3^Z$ in the dimensional regularization with anti-commuting $\gamma_5(\{\gamma_\mu, \gamma_5\} = 0)$, which obviously respects gauge symmetry. As for f_5^Z, the usual dimensional method gives vanishing f_5^Z. This is because of the identity $Tr\ (\gamma_\mu\gamma_\alpha\gamma_\beta\gamma_\rho\gamma_5) = 0$, the same reason that the chiral anomaly cannot be obtained in this regularization. We have used alternative regularizations, with the four-dimensional γ_5, $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$. After the

contributions of $U,\ D,\ N$ and E are summed up, the final result of f_5^Z turns out to be unique.

We have checked by an explicit calculation that in the limit of large fermion masses all form factors except the four vanish, as they should, in particular $\bar{f}_1^\gamma = 0$ and $f_5^\gamma = (-p^2)h^\gamma$. The results on the remaining four parameters are listed below:

$$
\begin{aligned}
\bar{f}_1^Z &= -\frac{\alpha}{32\pi s^2 c^2}\sum_{i=Q,L} N_i\Big\{\frac{m_{U_i}^4 + m_{D_i}^4 - 6m_{U_i}^2 m_{D_i}^2}{(m_{U_i}^2 - m_{D_i}^2)^2} + \frac{2m_{U_i}^2 m_{D_i}^2 (m_{U_i}^2 + m_{D_i}^2)}{(m_{U_i}^2 - m_{D_i}^2)^3}\ \ln\frac{m_{U_i}^2}{m_{D_i}^2}\Big\}\ , \\
\bar{f}_3^\gamma &= -\frac{\alpha}{24\pi s^2}\sum_{i=Q,L} N_i\Big\{\frac{m_{U_i}^4 + m_{D_i}^4 - 14m_{U_i}^2 m_{D_i}^2}{2(m_{U_i}^2 - m_{D_i}^2)^2} + \frac{3m_{U_i}^2 m_{D_i}^2 (m_{U_i}^2 + m_{D_i}^2)}{(m_{U_i}^2 - m_{D_i}^2)^3}\ln\frac{m_{U_i}^2}{m_{D_i}^2}\Big\} \\
&\quad -\frac{\alpha}{8\pi s^2}\sum_{i=Q,L} N_i Y_i\Big\{\frac{m_{U_i}^2 + m_{D_i}^2}{m_{U_i}^2 - m_{D_i}^2} - \frac{2m_{U_i}^2 m_{D_i}^2}{(m_{U_i}^2 - m_{D_i}^2)^2}\ln\frac{m_{U_i}^2}{m_{D_i}^2}\Big\}\ , \\
\bar{f}_3^Z &= -\frac{\alpha}{96\pi s^2 c^2}\sum_{i=Q,L} N_i\Big\{\frac{5m_{U_i}^4 + 5m_{D_i}^4 - 46m_{U_i}^2 m_{D_i}^2}{(m_{U_i}^2 - m_{D_i}^2)^2} \\
&\qquad + \frac{18m_{U_i}^2 m_{D_i}^2 (m_{U_i}^2 + m_{D_i}^2)}{(m_{U_i}^2 - m_{D_i}^2)^3}\ln\frac{m_{U_i}^2}{m_{D_i}^2}\Big\} - \left(\frac{s^2}{c^2}\right)\bar{f}_3^\gamma\ , \\
f_5^Z &= -\frac{\alpha}{32\pi s^2 c^2}\sum_{i=Q,L} N_i\Big\{\frac{m_{U_i}^2 + m_{D_i}^2}{m_{U_i}^2 - m_{D_i}^2} - \frac{2m_{U_i}^2 m_{D_i}^2}{(m_{U_i}^2 - m_{D_i}^2)^2}\ln\frac{m_{U_i}^2}{m_{D_i}^2}\Big\}\ ,
\end{aligned}
\tag{16}
$$

where $\alpha \equiv e^2/(4\pi)$ and $1 - c^2 = s^2 \equiv \sin^2\theta_W$, and $m_{U_Q} = m_U$ and $m_{U_L} = m_N$, etc..

As an illustrative example we consider an extreme case,

$$
m_U^2 \gg m_D^2\ \ ,\quad m_N^2 \gg m_E^2\ \ . \tag{17}
$$

It is interesting to note that the logarithmic dependence disappears, i.e.,

$$
\begin{aligned}
\bar{f}_1^Z &\simeq -\frac{\alpha}{32\pi s^2 c^2}(3+1)\ \ , \\
\bar{f}_3^\gamma &\simeq -\frac{\alpha}{24\pi s^2}(3-1)\ \ , \\
\bar{f}_3^Z &\simeq -\frac{\alpha}{96\pi s^2 c^2}[3\times(5-4s^2) + (5+4s^2)]\ \ , \\
f_5^Z &\simeq -\frac{\alpha}{32\pi s^2 c^2}(3+1)\ \ ,
\end{aligned}
\tag{18}
$$

where the first term in the parentheses is the $U,\ D$ contribution, while the second that

of $N,\ E$.

(b) *Technihadrons*

We have made a rough estimate of the contributions of technihadrons to the four parameters by taking the free technifermion picture and therefore applying eq.(16). The technifermions are assumed to have large constituent masses of order Λ_{TC}. Their masses are taken to be degenerate, $m_{T_U} = m_{T_D}$ and $m_{T_N} = m_{T_E}$, in accord with the custodial symmetry in the technicolor condensation, $< \bar{T}_U T_U >=< \bar{T}_D T_D >$, $< \bar{T}_N T_N >=< \bar{T}_E T_E >$. This picture is suggested by analogy to the result in QCD that the local average of the R-ratio due to hadrons is well described by free quark contributions. For the one generation technicolor model (with N_{TC} technicolors) we have obtained

$$\begin{aligned}
\bar{f}_1^Z &= -\frac{\alpha}{6\pi s^2 c^2} N_{TC} \quad , \\
\bar{f}_3^\gamma &= -\frac{\alpha}{6\pi s^2} N_{TC} \quad , \\
\bar{f}_3^Z &= -\frac{\alpha}{6\pi s^2 c^2}(2 - s^2) N_{TC} \quad , \\
f_5^Z &= 0 \quad ,
\end{aligned} \tag{21}$$

where the f_5^Z, being antisymmetric under the exchange of $m_U \leftrightarrow m_D, \quad m_N \leftrightarrow m_E$, vanishes under the exact custodial symmetry.

To get a rough idea of the magnitude of the effects of heavy particles in these four parameters, we take the model of one heavy fermion generation as an example. For large m_U^2/m_D^2 and m_N^2/m_E^2 we have from eq.(18) that $\bar{f}_1^Z = -\alpha/(8\pi s^2 c^2) \simeq -0.002$, which is comparable to the values of S parameter, $\alpha S = 2\alpha/(3\pi) \simeq 0.002$ (for $m_U = m_D$ and $m_N = m_E$). The experimental determination of S is reaching the precision of this level. As for the four parameters, it seems difficult to detect the effects of this minute size in the first generation experiments of $e^+e^- \to W^+W^-$.

4. Summary and remarks

Any beyond-the-standard model (new physics) contains heavy particles of characteristic mass M much larger than M_W. In some of such models heavy particles manifest themselves in light particle processes through radiative corrections; their effects do not decouple in the limit of large M^2/M_W^2. We have studied the non-decoupling effects in the triple gauge boson (TGB) vertices γW^+W^- and $Z^0W^+W^-$. We have sorted

out four parameters, $\bar{f}_1^Z$, $\bar{f}_3^\gamma$, $\bar{f}_3^Z$, f_5^Z, which describe the non-decoupling effects, in comparison with three parameters, (S, T, U), in the gauge boson two-point functions.

One may wonder whether there exists some relation between the non-decoupling effects of gauge boson two-point functions and those of three-point functions. As for the heavy particle contributions of type i) (see the beginning of this paper), the coefficient of the higher dimensional ($d > 4$) operators is inversely proportional to the $SU(2)_L \times U(1)$ singlet large mass M_S. Hence $d = 6$ operators are the most important ones and the two- and three-point functions are expected to be mutually related [9].

In the type ii) case of heavy particle effects, we claim that the four parameters $\bar{f}_1^Z$, $\bar{f}_3^\gamma$, $\bar{f}_3^Z$, f_5^Z and the three parameters S, T, U are independent. This can be demonstrated by noting that there exists a set of seven independent (non-decoupled) operators consisting of $D_\mu \phi$ and $\mathbf{W}_{\mu\nu} \equiv \frac{1}{2}\sigma^a W^a_{\mu\nu}$:

$$
\begin{aligned}
&(\phi^\dagger D_\mu \phi)^2 \;, \quad (\phi^\dagger \mathbf{W}_{\mu\nu} \phi) B^{\mu\nu} \;, \quad i\ (D_\mu\phi)^\dagger (D_\nu \phi) B^{\mu\nu} \;, \\
&i\ (D_\mu \phi)^\dagger \mathbf{W}^{\mu\nu} (D_\nu \phi) \;, \quad (\phi^\dagger \mathbf{W}_{\mu\nu}\phi)^2 \;, \quad i\ (\phi^\dagger \mathbf{W}^{\mu\nu} \phi)\ (D_\mu\phi)^\dagger\ (D_\nu \phi) \;, \\
&\epsilon^{\mu\nu\rho\sigma}\ (\phi^\dagger D_\mu \phi)\ [\phi^\dagger \mathbf{W}_{\rho\sigma} D_\nu \phi - (D_\nu\phi)^\dagger \mathbf{W}_{\rho\sigma}\phi] \;,
\end{aligned}
\tag{22}
$$

modulo equations of motion. The seven parameters $\bar{f}_1^Z$, $\bar{f}_3^\gamma$ $, \cdots, U$ are expressed in terms of (linear combinations of) the coefficients of the seven operators (22) appearing in the effective Lagrangian.

In our computation of the TGB couplings we have made no assumption as to the Higgs mass m_H, and hence our result holds true for arbitrary values of m_H. If m_H is light, the physical Higgs field ϕ appears in the non-decoupled operators listed in (22). If one restricts one's consideration to the extreme limit of $m_H \to \infty$, one can develop a different formalism eliminating ϕ. In the case of custodial isospin symmetry and parity conservation (the technicolor model is an example), one can use the gauged non-linear σ-model to examine the problem we studied above [13]. The task is to find all possible operators of $d \leq 4$ in terms of the dimensionless field $U = \exp(iG^a\sigma^a/2v)$ (G^a are the would-be Goldstone modes) and the covariant derivative D_μ. The argument has been extended by incorporating isospin violation effects. Seven operators are identified in this approach by Appelquist and Wu [14].

Since $\bar{f}_1^Z, \bar{f}_3^\gamma, \bar{f}_3^Z, f_5^Z$ are all dimensionless functions they do not blow up even when some of heavy particle masses become very large, in clear contrast to the case of the

T parameter. The magnitudes of the four parameters are comparable to that of αS. Since the four parameters are independent of the oblique correction parameters, testing beyond-the-standard models by measuring these four parameters in $e^+e^- \to W^+W^-$ experiments is important. In practice, precise measurements of them in the coming first generation experiment do not appear promising.

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Figure Captions

Fig. 1 The t-channel (ν-exchange) and the s-channel (V-exchange with $V = \gamma$ or Z^0) graphs contributing to the process $e^+e^- \rightarrow W^+W^-$. The s-channel graph has three types of corrections due to heavy particle loops, the oblique correction to the V propagator, the VW^+W^- vertex correction and the $W^\pm$ leg correction, as indicated by blobs in the figure.